

HEAT TRANSFER AND ICE-MELTING IN AMBIENT WATER NEAR ITS DENSITY EXTREMUM

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(Received 30 March 1976)

Abstract—An experimental study of natural convection flow over a vertical ice slab immersed in cold water has been carried out. The effect of changing the ambient water temperature, on heat transfer, was measured. The major emphasis was on ambient temperatures in the vicinity of the density extremum of water. The effect of the density extremum on flow direction was investigated and the ambient temperature where upflow changes to downflow was determined. The experimentally determined heat-transfer results were compared with theoretical results giving a mean magnitude of deviation of 5.6%

NOMENCLATURE

- Gr , Grashof number;
- \bar{h} , average convection coefficient;
- h_{il} , latent heat of fusion;
- I_w , net buoyancy force;
- L , ice-slab height;
- \dot{m} , surface melting rate;
- \dot{M} , instantaneous weight reading;
- Nu , Nusselt number;
- Pr , Prandtl number;
- q , density temperature dependence;
- R , density range parameter;
- w , ice-slab width;
- W , local buoyancy force.

Greek symbols

- α , density temperature dependence;
- β , coefficient of thermal expansion;
- ρ , density;
- ϕ , temperature excess ratio.

Subscripts

- ∞ , ambient condition;
- i , ambient condition which causes flow inversion;
- 0 , surface condition;
- f , film condition;
- m , at the density extremum.

INTRODUCTION

NATURAL convection flows along vertical surfaces in an extensive uniform ambient medium have been studied extensively in the past. Most considerations have been based on the Boussinesq approximation to a monotonic density-temperature relationship. This type of analysis does not apply accurately for cold water, one of the most common fluid states occurring

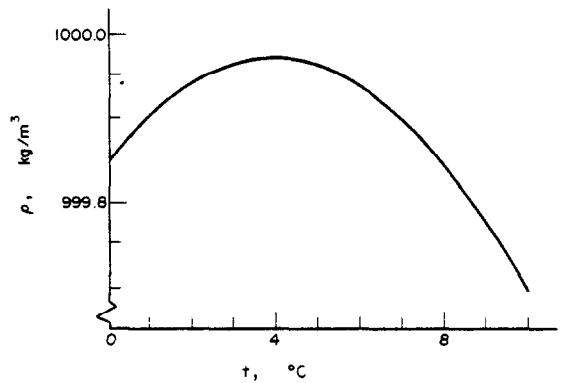


FIG. 1. A density-temperature profile for water at 1 atm abs, calculated from Kell [1].

in nature. There is a density extremum, at around 4°C at 1 atm for pure water. A distribution is seen in Fig. 1, calculated from Kell [1].

Codegone [2], in 1939, was apparently the first to demonstrate convective reversals or "inversions" around the density extremum. However, the first detailed heat-transfer measurements in water, under conditions of density inversion, were apparently made by Ede [3], in 1951, on a 10.2 cm high electrically heated plate. These results are summarized in a letter. Ede [4], written in response to the analysis of Merk [5].

Merk [5] was perhaps the first to analyse the effect of the density extremum on natural convection by calculating its effect on the heat transfer of a melting sphere of ice in cold water, using integral method of analysis. He predicted a convective inversion at 5.31°C, while 4.8°C was obtained experimentally by Dumore *et al.* [6]. Vanier and Tien [7] also did an experiment with melting ice spheres and obtained a convective inversion at 5.35°C.

The data of Ede was in reasonable agreement with

Merk's calculations. The mean magnitude of the deviations was 9.9%. Vanier and Tien [8] then numerically calculated the vertical isothermal flat plate natural convection problem for various wall and ambient temperature combinations, by finding similarity using a cubic polynomial representation of density. These results were compared with certain of Ede's and gave agreement with a mean magnitude of deviation of 3%. However, several areas near the convective inversion were designated as zones of no solution, as no numerical convergence occurred.

Schechter and Isbin [9] used a vertical aluminum plate with four separate heaters to obtain an isothermal condition. The surrounding water was cooled down to the range where the density extremum is important. Qualitative observations on the dual nature of the flow, as well as flow direction, were reported. Velocity and temperature distributions, as well as transport rates, were measured and compared with analogue computer results, using Merk's density expression for water.

Tkachev [10] carried out experiments with vertical and horizontal ice cylinders immersed in cold water. He found a minimum Nusselt number at $t_{\infty} = 5.5^{\circ}\text{C}$, for the vertical cylinder.

The present experimental study deals with a vertical ice slab immersed in water at a uniform bulk ambient temperature, t_{∞} . The effect of changing t_{∞} on heat-transfer rates and on flow direction is considered. The coefficient of thermal expansion, β , changes sign at the extremum, as seen in Fig. 1. However, the flow may be up or down, independent of β , depending on the values of t_0 and t_{∞} . For example, if $t_0 = 0^{\circ}\text{C}$ and $t_{\infty} = 20^{\circ}\text{C}$, the fluid in the boundary layer is more dense than in the ambient medium and the convection currents are down. However, if $t_0 = 0^{\circ}\text{C}$ and $t_{\infty} = 2^{\circ}\text{C}$, the fluid in the boundary layer is less dense, and the flow is up.

In both of these two examples, the wall was colder than the ambient fluid. However, they resulted in different flow directions. Also, the first example, in which $t_{\infty} = 20^{\circ}\text{C}$, is not as simple as a first glance might suggest. Part of the fluid is locally more dense, while some of the fluid is locally less dense. The important quality in determining the flow direction is perhaps the net density difference summed across the boundary region. Note that for $t_{\infty} = 6^{\circ}\text{C}$, the buoyancy force reverses across the region near the surface.

For a given t_0 , the ambient temperature, t_{∞} , where a slightly higher t_{∞} will result in net flow in one direction, while a slightly lower t_{∞} will result in net flow in the opposite direction, is called the inversion temperature, t_i . A reversal of flow direction is accommodated in the analysis by simply reversing the positive direction of the vertical coordinate, x .

When t_{∞} is close to t_i for a wall temperature, t_0 , of 0°C , the convective currents are very slow, they are characterized by a small Grashof number. Small Grashof numbers nominally invalidate the use of the first order boundary layer approximations used, e.g. by Vanier and Tien [8] and by Gebhart and Mollendorf [11]. In addition, each of these analyses encountered

a region where numerical solutions could not be easily obtained. This region corresponded to conditions around t_i , where local flow reversal may occur, in contrast to complete flow inversion. The results of this experimental study are compared with the calculation of Gebhart and Mollendorf [11]. They also fill in the range very near the flow inversion.

THE EXPERIMENT

The experiment was carried out in the melting of a 30.3 cm high and 14.8 cm wide ice slab which was initially 3 cm thick. The ice was surrounded by a picture frame arrangement of 2.5 cm thick expanded polystyrene insulation. An acrylic plastic frame around the insulation added strength. Embedded in the ice were three 0.13 mm copper-constantan thermocouples, to indicate when the ice initially was essentially at 0°C , before the test run. Thus, there was no heat conduction interior to the ice slab. Two stainless steel weights of 504 g each were hung from the frame to counteract the buoyancy of the submerged ice slab and accompanying frame. See Fig. 2.

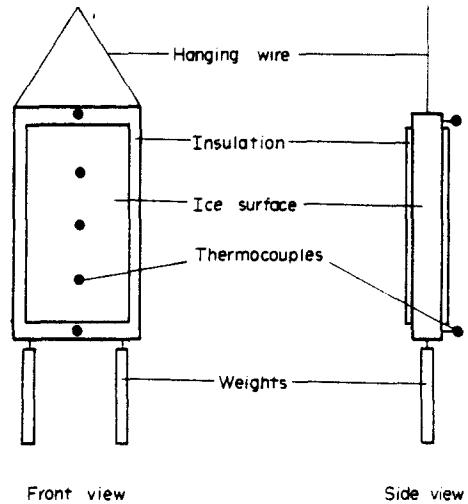


FIG. 2. The ice-slab arrangement.

The slab was suspended from a 21 gauge teflon coated wire connected to a hook assembly inside of a Sartorius model 2257 balance. The precision of this balance is ± 0.008 g with a 0.01 g readability rating. Using this balance, with a Beckman digital timer, the melting rate of the ice slab was determined for each ambient tank temperature, t_{∞} , desired.

The bulk tank temperature was adjusted by a 1000 Btu/h (293 W) circulating chiller. Ethylene glycol was pumped from the chiller through a coiled stainless steel tubing heat exchanger immersed near the bottom of a tank, filled with 0.265 m³ of distilled water. The tank was stainless steel, 0.86 m high by 0.61 m by 0.61 m, insulated with 5 cm of expanded polystyrene, which was covered by plywood. The tank had two 20 cm dia acrylic windows for observations. These were usually covered with 7 cm of insulation.

The ambient water temperature was made uniform by stirring the tank contents at frequent intervals

preceding a run. The stirrer was stopped at least 0.5 h preceding the start of each experimental run, to reach a quiescent state. Five 0.13 mm copper-constantan thermocouples, each one 12.7 cm above the next lower one, were present to measure the vertical temperature gradient in the ambient medium. These thermocouples were connected differentially, with reference to the lowest junction, through a switch box to a Beckman type R Dynograph. During the experiment, vertical gradients of $0.2^{\circ}\text{C}/\text{m}$ were typical. A precision mercury-in-glass thermometer, with 0.1°C divisions, was placed midway between the top and bottom of the tank, to measure the ambient water temperature, t_{∞} .

frame, with the junctions extending out into the ambient medium. One was in the top horizontal portion of the insulating frame and the other was in the bottom, see Fig. 2. They were connected differentially and monitored throughout an experiment to determine whether the flow was up or down. They were at positions 1 cm above and below the top and bottom, respectively, of the edge of the ice slab. When the flow was up, the top thermocouple was colder than the bottom one, as the warmer t_{∞} was entrained past the bottom thermocouple while the colder convective wake flowed over the top thermocouple. For down flow, the effect was reversed.

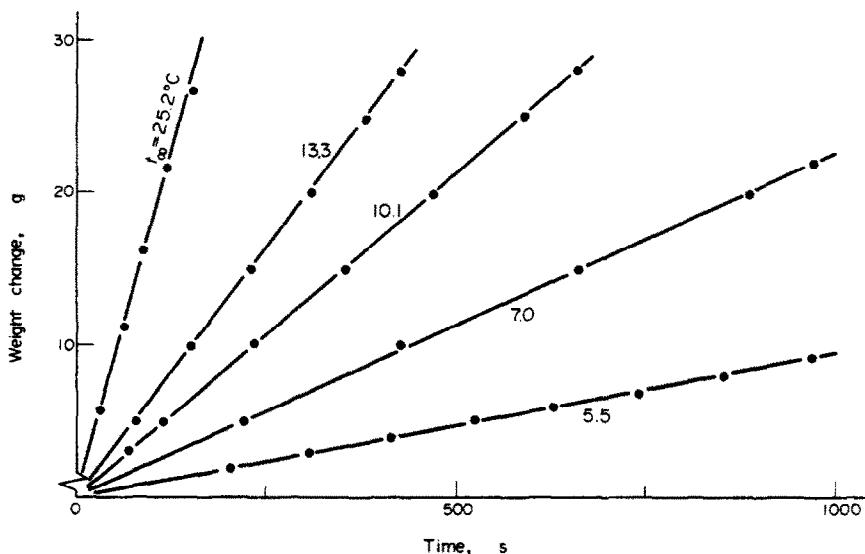


FIG. 3. Changes in the weight of melting ice-slabs in water.

The ice slabs were made in a horizontal position by freezing distilled water in an aluminum mold. The mold was insulated on the four edges, with the top and bottom uninsulated. This arrangement caused freezing of the ice in from the top and bottom surfaces of the slab. As a result any dissolved gases coming out of solution formed bubbles near the mid-plane of the slab, where they were not released during melting. If ice with entrained air bubbles were melted, the bubbles would introduce an inaccuracy in the weight-determined melting rate. The rising bubbles would also drag fluid with them, affecting the convective currents under study, and, thereby, the resulting heat-transfer rate. To further decrease the formation of bubbles, the water which was to go into the mold was first vigorously boiled. Soon after boiling the water, the molds were placed in a freezer at -20°C .

The mold for forming the ice was such that the resulting slab of ice was 4.6 mm thicker than the frame of insulation. Thus, the insulation was initially slightly recessed with respect to the melting surface, see Fig. 2. All ice was removed from the surrounding insulation to provide a well defined area of ice-water interface.

Two thermocouples were positioned in the insulating

EXPERIMENTAL RESULTS

An experimental run consisted of an ice slab, initially at 0°C throughout, immersed at a preset t_{∞} . The instantaneous weight readings, \dot{M} , were recorded through time. A plot determined the melting rate, \dot{m} . Typical weight determinations are shown in Fig. 3. The straight line which was thought to be most representative of the data was drawn through the points. In a 1200 s run, curvatures amounting to a 1% difference in the interpretation of the melting rate \dot{m} occurred.

The slope of the weight readings vs time was converted to a melting rate, \dot{m} , by accounting for the buoyancy of ice in water and using the density of ice as 916.8 kg m^{-3} , as follows:

$$\dot{m} = \frac{\rho(t_{\infty})}{\rho(t_{\infty}) - \rho_{\text{ice}}} \dot{M}. \quad (1)$$

The density of water was determined from t_{∞} , using Kell's polynomial expression for pure water at 1 atm.

By knowing the melting rate of ice, \dot{m} , at a particular t_{∞} , the Nusselt number may be determined, using the latent heat of fusion, $h_{if} = 79.77 \text{ cal/g}$, and taking into account the area of a two-sided slab. We assume that

the conduction of heat into the slab is negligible, since the slab was initially equilibrated in air until the center temperature was within 0.1°C of 0°C . The Nusselt number is evaluated as

$$Nu_L = \frac{\bar{h}L}{k} = \frac{\dot{m}h_{if}}{2wk(t_x - t_0)} \quad (2)$$

where $L = 30.3$ cm is the height of the ice surface, \bar{h} is the average heat-transfer coefficient and $w = 14.8$ cm is the width of the ice slab. Thermal conductivity is evaluated at the "film" temperature,

$$t_f = (t_0 + t_x)/2. \quad (3)$$

In Fig. 4 the experimental Nusselt numbers are plotted, as points, vs t_x . When $t_x = 5.6^\circ\text{C}$, downflow was observed, while at $t_x = 5.5^\circ\text{C}$, upflow was observed. A Nusselt number of 38.9 was the minimum value found. It occurred at $t_x = 5.6^\circ\text{C}$.

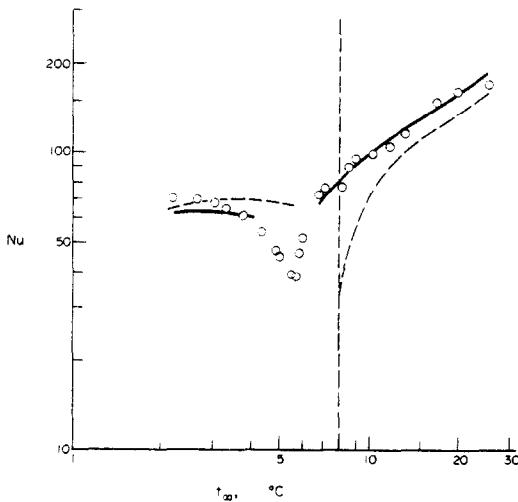


FIG. 4. The experimental results are shown as points, at various values of t_x . The solid curve is from the theory and analysis of Gebhart and Mollendorf [11]. The dashed curve is the prediction with the Boussinesq approximation.

The present experimental results will be compared with preliminary information from the analysis and numerical results of Gebhart and Mollendorf [11], for a vertical, flat, semi-infinite, isothermal, non-melting plate, with t_x in the vicinity of a density extremum. The density equation used in the analysis was the preliminary form of that developed by Gebhart and Mollendorf [12] for both pure and saline water to 1000 bars and to 20°C . For pure water at 1 bar abs, the final form reduces to the following simple result:

$$\rho(t) = \rho_m(1 - \alpha|t - t_m|^q) \quad (4)$$

where the density is a maximum of $\rho_m = 999.9720$ kg m^{-3} at the temperature $t_m = 4.0293^\circ\text{C}$, $\alpha = 9.297173 \times 10^{-6}$ and $q = 1.894816$. With these values (4) agrees with the correlation of Kell to about 2.00 ppm to 20°C . This density expression, equation (4), was used to calculate the buoyancy force, $\rho_\infty - \rho$, instead of using the following Boussinesq approximation

$$\rho_\infty - \rho = \rho\beta(t - t_\infty). \quad (5)$$

Equation (4) leads to a buoyancy force, in Gebhart and Mollendorf's similarity analysis, of

$$W = |\varphi - R|^q - |R|^q \quad (6)$$

where

$$R = \frac{t_m - t_x}{t_0 - t_x} \quad (7)$$

and

$$\varphi = \frac{t - t_x}{t_0 - t_x}. \quad (8)$$

The parameter R defined in equation (7) represents the relative effect of the density extremum. For example, if $t_0 = 20^\circ\text{C}$ and $t_x = 24^\circ\text{C}$, then $R = 5$. For these temperatures one might expect a small effect of the density extremum on the flow and this is characterized by a large value of R . However, the density extremum is important for values of R which are small in magnitude, for example, $R = 0$ for $t_x = t_m$, an obvious condition in which the effect of the density extremum is decisive.

The local buoyancy force, W , defined in equation (6), is dependent upon R . If $R \geq \frac{1}{2}$, then W is negative throughout the boundary region, except for $R = \frac{1}{2}$, when it is zero at the interface. Thus, the buoyancy force is down and the flow is down. Likewise, when $R \leq 0$, W is positive throughout the boundary region and the flow is up. In the range between $0 < R < \frac{1}{2}$, at $R = 0.1$ for example, the local buoyancy force near the surface is positive and becomes negative further away. The question of the resulting net flow direction may perhaps be resolved by considering the net buoyancy force across the boundary layer, I_w , defined by Gebhart and Mollendorf [11], as

$$I_w = \int_0^x W(\eta) d\eta. \quad (9)$$

A positive I_w suggests that the net buoyancy is up, while a negative I_w suggests down. An extrapolation of the results of Gebhart and Mollendorf [11] indicates that $I_w = 0$ at $R \approx 0.31$. This corresponds to $t_x = 5.8^\circ\text{C}$, for $t_0 = 0^\circ\text{C}$. Our experimentally determined inversion implies that $I_w = 0$ somewhere between t_x of 5.5°C ($R = 0.27$) and 5.6°C ($R = 0.28$).

The value of I_w not only indicates the sign of net buoyancy. It is also related to the magnitude, or vigor, of the flow. This is indicated by its presence in the Grashof number Gebhart and Mollendorf [11] used to obtain similarity, with the buoyancy force term as in equation (6).

$$Gr_x = \frac{g x^3 \alpha |t_0 - t_x|^q I_w}{\nu^2}. \quad (10)$$

Using this definition of the Grashof number, the Nusselt number is calculated to be:

$$Nu_L = \frac{1}{3} \left(-\frac{\varphi'(0)}{\sqrt{2}} \right) Gr_L^{1/4} = Nu_L(Gr_L, Pr, R) \quad (11)$$

where $\varphi'(0)$ is the slope of the temperature excess ratio at the surface-liquid interface. Equation (11) is identical

Table 1. Comparison of experimental results with the similarity analysis results of Gebhart and Mollendorf [11]

t_∞	R	Pr at t_f	Gr_L	Nu_L^\ddagger exp.	Nu_L^\ddagger theoretical	% Deviation
25.2	0.841	8.6	4.18×10^8	172.5	190.1	9.3
19.9	0.798	9.3	1.73×10^8	163.4	157.9	3.5
16.9	0.762	9.8	1.06×10^8	149.1	142.5	4.7
13.3	0.697	10.3	5.18×10^7	116.9	122.8	4.8
11.7	0.656	10.6	3.55×10^7	105.2	112.8	6.8
10.1	0.601	10.9	2.10×10^7	98.4	101.2	2.8
9.1	0.557	11.1	1.45×10^7	96.4	93.6	3.0
8.5	0.526	11.2	1.07×10^7	90.6	88.0	3.0
8.0	0.496	11.3	8.21×10^6	77.5	82.8	6.3
7.0	0.424	11.5	3.96×10^6	76.8	71.6	7.4
6.8	0.407	11.7	3.08×10^6	72.4	68.3	6.0
6.0	0.328	11.7	5.77×10^5	52.1		
5.8	0.305	11.7	—	46.7		
5.6	0.280	11.8	5.54×10^5	38.9		
5.5	0.267	11.8	7.63×10^5	39.4		
5.0	0.194	11.9	1.87×10^6	45.9		
4.9	0.178	11.9	2.21×10^6	47.6		
4.4	0.084	12.0	2.86×10^6	55.8		
4.0	-0.007	12.1	3.36×10^6	60.0	60.5	0.9
3.3	-0.221	12.3	3.67×10^6	65.3	63.2	3.5
3.0	-0.343	12.3	3.58×10^6	68.2	63.4	7.6
2.7	-0.492	12.4	3.60×10^6	68.8	64.1	7.4
2.2	-0.832	12.5	3.23×10^6	70.5	62.8	12.3

*Nusselt number based on a plate length of 0.303 m; for other lengths, multiply by $(L/0.303)^{3/4}$.

† Results from calculations of Gebhart and Mollendorf [11].

to that which results with the Boussinesq approximation. However $-\phi'(0)$ is not the same, since the buoyancy force in that analysis, ϕ , is replaced by W above. The calculations of Gebhart and Mollendorf [11] give $\phi'(0)$ and I_w as functions of Prandtl number and of R . The results, calculated for variable Prandtl number across the range of t_f for the experiments, is plotted as the solid curve in Fig. 4.

The experimental results are also compared with the results of conventional analysis which results from using the Boussinesq approximation for the density difference, equation (5). These results are well known, as summarized, for example, by Gebhart [13]. The Nusselt number definition remains as in equation (11), with the appropriate $\phi'(0)$ found numerically for each Prandtl number. The Grashof number becomes,

$$Gr_{L,B} = \frac{g\beta L^3 |t_0 - t_\infty|}{\nu^2} \quad (12)$$

The resulting Nusselt number variation with t_∞ is also plotted in Fig. 4, as the dashed curve. All properties, including β , were evaluated at t_f , equation (3).

As a result of evaluating β at t_f , the Nusselt number in the range $5.6^\circ\text{C} < t_\infty < 8^\circ\text{C}$ must be omitted from Fig. 4. In this range of t_∞ the value of β_f is negative, as is $(t_0 - t_\infty)$, indicating upflow, conventionally. However, the flow is known, from both experiment and calculation, to be down. If another criterion is used for evaluating β , the unreasonable range is merely shifted in t_∞ , not eliminated. For example, if β was evaluated

at t_∞ , then the Nusselt number is zero at 4°C , see equation (12). For $4^\circ\text{C} < t_\infty < 5.6^\circ\text{C}$, β will be positive although downflow actually occurs. Finally, if β is evaluated at 0°C , the surface temperature, then β will be negative for $t_\infty > 5.5^\circ\text{C}$, when the actual flow is down. For any choice of reference temperature for the evaluation of β , the Boussinesq model fails in the vicinity of the inversion point, t_i .

Instead of using t_f to determine k in the experimental Nusselt number, one might instead choose either t_0 or t_∞ . In the experimental run having the largest difference in $t_0 - t_\infty$, at $t_\infty = 25.2^\circ\text{C}$, k varies from $0.569 \text{ W/m}^2\text{C}$ at 0°C to $0.611 \text{ W/m}^2\text{C}$. This is a Nusselt number change of only 4%. For more typical experimental values of $t_0 - t_\infty$, the effect is $\pm 1\%$.

For the numerical calculations of Gebhart and Mollendorf [11], the effect of property reference temperature also arises. Recall that I_w and $\phi'(0)$ are functions of the Prandtl number, $c_p k/\mu$. These three properties interact. The effect will be assessed for the run at $t_\infty = 8.5^\circ\text{C}$. The value of Pr calculated at t_∞ is only 2% less than that calculated at t_f . Using t_0 results in a value only 2% higher. The greatest difference arising over the whole temperature range is only 7%.

Over the t_∞ temperature range of $25-0^\circ\text{C}$, the Prandtl number changes from 6 to 13. The Prandtl number was here calculated, at each temperature, from a polynomial expression from Kell [1] for c_p . Polynomials for k and μ were taken from the *A.S.M.E. Steam Tables* [14], which claim accuracy of $\pm 2\%$.

CONCLUSION

In many natural convection flows, the ambient temperature, t_x , is principally important only in relation to the surface temperature, t_0 , in a temperature difference. However, around a density extremum, the values of t_0 and t_x are both decisively important, in their relation to t_m .

A thermal diffusion caused variation of density, with an extremum, results in both flow and heat transfer which are even qualitatively different from usual behavior. For a surface temperature t_0 less than the temperature at the extremum, certain values of t_x cause upflow, while other values cause downflow.

We found, for a vertical ice surface, that upflow occurred when $t_x \leq 5.6^\circ\text{C}$. For $t_x \geq 5.5^\circ\text{C}$ downflow was observed, in close accord with some earlier results. In addition, the minimum Nusselt number in the experimental temperature range, $2.2^\circ\text{C} \leq t_x \leq 25.2^\circ\text{C}$, was found to occur at $t_x = 5.6^\circ\text{C}$.

Experimental Nusselt number values were near those predicted by a theoretical formulation, as seen in Table 1. No comparison is made in the immediate neighborhood of the flow direction inversion point, t_i , since calculations were not successful there. Very slow flows then exist, with the effective Grashof number becoming very small as t_x goes toward t_i . Therefore, the validity of simplest boundary-layer theory becomes questionable in this region. More realistic analysis would be much more difficult. No theoretical results were found in the literature for this regime.

Acknowledgement—The authors wish to acknowledge support by the National Science Foundation under grants ENG75-05466 and ENG75-22623 for this research. We also record our indebtedness to Dr. R. G. Bill, Jr., Mr. R. Mahajan, Mr. H. Shaikatullah and Dr. J. C. Mollendorf for valuable advice and help.

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TRANSFERT DE CHALEUR ET FUSION DE LA GLACE DANS DE L'EAU,
AUTOUR DU POINT DE DENSITE MAXIMALE

Résumé—On présente une étude expérimentale de l'écoulement en convection naturelle autour d'une plaque de glace verticale immergée dans de l'eau froide. L'effet d'un changement de température ambiante de l'eau sur le transfert de chaleur a été mesuré. Une importance particulière a été donnée aux températures ambiantes voisines du point de densité maximale de l'eau. On a étudié l'effet de la densité maximale sur la direction de l'écoulement et on a déterminé la valeur de la température ambiante pour laquelle l'écoulement ascendant devient descendant. Les résultats des mesures du transfert de chaleur comparés aux résultats théoriques présentent un écart moyen de 5.6 pour cent.

DER WÄRMEÜBERGANG UND DAS SCHMELZEN VON EIS IN
WASSER IN DER NÄHE SEINES DICHTEMAXIMUMS

Zusammenfassung—Es wurde eine experimentelle Studie der natürlichen Konvektionsströmung an einer vertikalen, in kaltes Wasser eingetauchten Eisplatte durchgeführt. Es wurde der Einfluß der Aenderung der Temperatur des umgebenden Wassers auf den Wärmeübergang gemessen. Eine besondere Beachtung erfuhr hierbei der Temperaturbereich in der Nähe des Dichtemaximums von Wasser. Es wurde der Einfluß des Dichtemaximums auf die Strömungsrichtung untersucht und die Temperatur bestimmt, bei welcher die Aufwärtsströmung in eine Abwärtsströmung umschlägt. Die experimentell ermittelten Werte für den Wärmeübergang wurden mit theoretisch berechneten Werten verglichen, wobei sich eine mittlere Abweichung von 5,6% ergab.

ТЕПЛООБМЕН И ТАЯНИЕ ЛЬДА В ВОДЕ ВБЛИЗИ ЕЕ ЭКСТРЕМУМА ПЛОТНОСТИ

Аннотация — Выполнено экспериментальное исследование естественной конвекции вдоль вертикальной плиты льда, погруженной в холодную воду. Измерялось влияние изменения температуры воды на перенос тепла. Основное внимание обращалось на температуру воды вблизи ее экстремума плотности. Исследовалось влияние экстремума плотности на направление потока и определена температура окружающей среды там, где восходящий поток изменялся на нисходящий. Экспериментальные данные по теплообмену сравнивались с теоретическими результатами, средняя величина отклонения не превышала 5,6%.